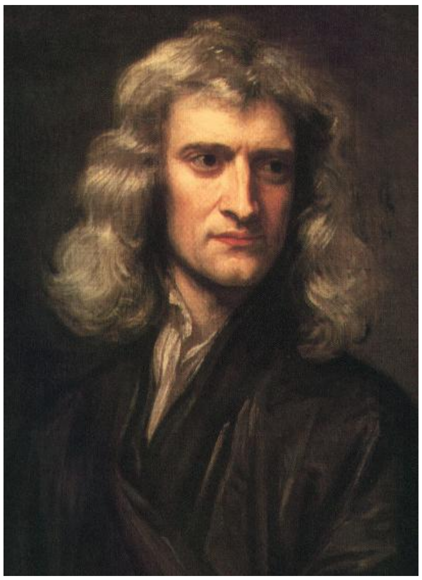

Atwood's Machine: Verifying Newton's Second Law

Author: K. R. Sturgeon
Partner: I. LuvFisics

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Danville Area Community College
PHYS 106

643-1727

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Abstract

The purpose of this lab is to verify Newton's Second law which is a statement relating force, mass, and acceleration. Newton's Second law states that acceleration (a) is directly related to net force (F_{Net}) and inversely related to mass (m). Thus, $a = \frac{F_{Net}}{m}$. Using the Atwood's Machine (see Appendix C), an experimental acceleration can be measured and compared to the theoretical acceleration which is predicted by Newton's Second law and is dependent on only two mass measurements and the known value of acceleration due to gravity. If these values match then the Second Law holds true. If the values do not match then the Second law is not valid. It is expected that the Second law will hold true and indeed, the results of this law confirmed Newton's Second Law within a percent error of 2.19%.

Introduction & Discussion

We embark upon the journey of verifying Newton's Second Law. This law is essential in our understanding of the world around us. We use Newton's Law to build bridges, understand our solar system, analyze car accidents, and launch space shuttles. These applications are only a few uses of the Second Law. Many more follow, as Newton's Law is actually used by physicists as a standard for all other theories. For example, Maxwell's Equations were accepted as a valid theory because they reduced to Newton's Law of Motion (Serwey, 2004).

The study of motion had its founding with Aristotle who claimed that motion could be divided into two classes: natural and violent. In his view every object has a determined nature. For example, heavier objects would "strive more" meaning they would fall faster. He also believed that objects have a natural state of rest and thus would not move unless a push or pull was acting upon it (Hewitt, 2002). Galileo Galilei (1564-1642) was an essential person in the development of the laws of motion, because his experimental work with incline planes led him to believe that objects would remain in motion unless an outside force existed. Newton, who was born the same year that Galileo died, built on Galileo's ideas by applying the mathematical understanding. Their work banished the Aristotle notions of motion almost completely (Galileo [online], 2004).

Newton's final result states, *"The acceleration of an object is directly proportional to the net force acting on the object, is in the direction of the net force, and is inversely proportional to the mass of the object. (Hewitt, 2002)."* This law is more commonly stated as $F_{Net} = ma$, where F_{Net} is the net force, m is the mass of the system, and *a* is the acceleration of an object. Net force, measured in Newtons (N), is the sum of all forces acting on an object, and acceleration is the rate at which the speed of an object is changing and is measured in units of m/s^2.

To verify this law, it will be used to predict an acceleration which will then be measured experimentally using an Atwood's Machine. The Atwood's machine is a computerized two mass pulley system with a laser eye as depicted in Appendix C. It is attached to Smart Pulley Software developed by PASCO. The software will make a direct measurement of acceleration by measuring the change in time that each individual spoke of the pulley blocks the laser sensor. If the law of motion is correct the experimental and theoretical values should be within 5% of each other. The theoretical value is obtained as follows:

Given that Newton's second law indicates that

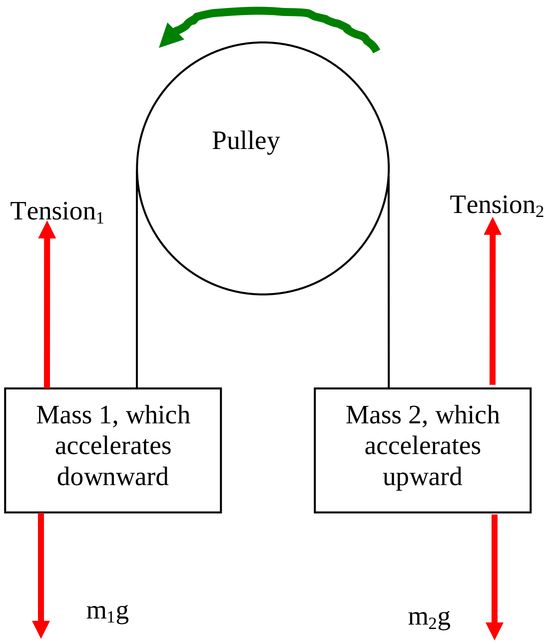
$$(1)\ F_{Net} = ma,$$

we must solve for the acceleration (a) and recognize that the mass is the total mass of the system meaning the sum of the hanging masses m_1 and m_2. The result is

$$(2)\ a = \frac{F_{Net}}{m_1 + m_2}$$

It is essential to obtain the net force, and thus, we must look at a force diagram for the system (see Figure 1). The system consists of two masses which are hanging from a pulley by a string. In Figure 1, the forces are depicted in red, and the direction of motion is shown in green.

Figure 1: Free Body Diagram



Net force is the sum of all forces within the free body diagram. The direction of motion will be considered the positive direction, thus, any forces pointing along the direction of motion will be positive and any pointing antiparallel to the direction of motion will be considered negative. The net force is

(3) $F_{Net} = +m_1 g - Tension_1 + Tenson_2 - m_2 g$.

Because the tension in a string is known to be the same regardless of location in the string, the two tension forces will cancel leaving

(4) $F_{Net} = m_1 g - m_2 g$.

Combining equation (4) with equation (2) gives the theoretical acceleration which is based on Newton's Second Law. The final result is

(5) $a = \dfrac{m_1 g - m_2 g}{m_1 + m_2}$, where m_1 and m_2 are the masses of the hanging objects and are the only values that must be measured. The acceleration due to gravity, g, is a known constant of 9.82 m/s^2. Sample calculations can be found in Appendix B.

In order for this method of verification to be valid, it is assumed that the only the motion of the objects affect the acceleration of the system. Thus, the pulley's motion does not contribute or hinder the acceleration of the system. It is also assumed that no other external forces exist. So, air resistance between the pulley and string, and friction within the pivot do not hinder the system.

Procedure

Materials: PASCO's Atwoods Machine, Computer with PASCO Smart Pulley Software, String, Several Hanging Masses.

1. Set up the apparatus as shown in Appendix C. Use a piece of thread about 10 cm longer than the distance from the top of the pulley to the floor. Connect the Smart Pulley to your computer and start the Smart Pulley software. To run the software refer to the product manual.

2. Place about 100 grams of mass on one mass hanger and record the total mass as m_2. Be sure to include the mass hanger in the total mass. Place slightly more than 100 grams on the other hanger. Record this mass as m_1.

3. Move the heavier of the two masses upward until the smaller mass almost touches the floor. With the mass motionless, and the pulley positioned so that the red LED on top is OFF.

4. Release the heavier mass, which will fall downward, pulling the lighter mass up. Stop the pulley just before the heavier mass reaches the floor, and halt the timing process of the computer.

5. When the computer finishes converting the times, graph the velocity vs. time.

6. At the top of the graph you should see three numbers. They are:
   M=slope of the graph
   B=y-intercept
   R=correlation coefficient (how close to a straight line it is)
   Record the slope of the graph, which is the experimental acceleration. Its units are in meters/sec^2.

7. Change the relationship between m_1 and m_2 by removing mass from one hanger and adding it to the other. This allows you to change the net force without changing the total mass. Repeat steps 3-6 at least 5 times using different combinations of m_1 and m_2. Record all your data.

8. (Analysis of Data) Calculate the net force for each situation, which is the difference between the two masses times the acceleration of gravity: (m_2-m_1) g. Calculate the total mass for each trial, (m_1+m_2). Calculate the theoretical acceleration using Newton's Second Law: F_{net} = ma. Compare the actual acceleration with the theoretical acceleration by determining the percentage difference between the two (See Appendix B for equations).

Troubleshooting.

1. The ratio of the masses do affect the results be sure to use masses that vary from each other.
2. The incline of the track can decrease or increase the acceleration, so it is essential to level the track.
3. Never run the track without air flow. This action results in scratching of the track which increases the drag and thus decreases the acceleration.
4. Be cautious to create a mechanism to prevent clanging of masses and the cart.

Results

In an attempt to verify Newton's Second law of motion, the following data (see Table 1) was collected for five trials. The first column represents the individual trails. In each trial, two masses were connected via a string over a pulley. m_1 is the mass of the heavier object, and m_2 is the mass of the lighter object. The Smart pulley software for the Atwood's Machine measured the experimental acceleration and the theoretical acceleration was calculated (see Appendix B) using Equation 5 with the gravitational acceleration set equal to 9.82 m/s^2. The final column within Table 1 is the percent error which shares information regarding how close the experimental acceleration is to the theoretical acceleration. It is calculated using Equation (6) from Appendix B. Percent Errors $\leq$ 5% is considered acceptable.

Table 1: Results

Trial	***m_1 (g)***	***m_2 (g)***	***Theoretical Acceleration (m/s^2)***	***Experimental Acceleration (m/s^2)***	***% Error***
1	100	15	7.26	7.02	-3.3%
2	90	25	5.55	5.65	1.8%
3	75	40	2.13	2.14	0.05%
4	65	50	1.28	1.30	1.6%
5	60	55	0.42	.60	4.3%

You will notice from Table 1 that as the mass of the heavier object decrease the acceleration does too. This relationship indicates that the acceleration is directly related to the heavier mass and indirectly related to the lighter mass. Also, notice that the percent error is well within the calculated range of 5% with an average of 2.19% and thus, suggests that Newton's Second Law of Motion is indeed correct.

Summary

The purpose of the lab was to verify the Second Law of Motion which states that $a = \frac{F_{Net}}{m}$ where a is the acceleration, F_{Net} is the sum of all forces, and m is the mass of the system. Thus, Newton's Second Law indicates that the acceleration is directly related to net force and inversely related to mass. By comparing the predicated acceleration calculated with Newton's Second Law to the measured acceleration, it was successfully shown (within 2.19% percent error on average) that the Second Law of Motion was indeed verifiable and thus, true. So, acceleration is directly related to force and inversely related to mass.

The primary source for the difference in the values is most likely within the assumption that the pulley has no affect on the acceleration. Given that it will take energy to accelerate the pulley as well, it is highly possible that the pulley itself would actually decrease the acceleration of the system. This conclusion is indeed supported with the data as four out of five trials show an experimental acceleration less than the theoretical acceleration.

Appendix A: Work Cited

Galileo: the Telescope & The Laws of Dynamics [online]. Available: http://csep10.phys.utk.edu/astr161/lect/history/galileo.htm (2004, November 2)

Hewitt, P. G. (2002). Conceptual Physics Ninth Edition. San Francisco: Addisen Welsey.

Serwey, R. A. (2004). Physics for Scientists and Engineers Sixth Edition. Belmont, CA. Thomson Brooks/Cole.

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Appendix B: Sample Calculation

Theoretical acceleration was derived from Newton's Second Law and was calculated using equation (5)

$$(5)\quad a = \frac{m_1 g - m_2 g}{m_1 + m_2}.$$

Thus, using information from Trial 1, the acceleration can be calculated as

$$a = \frac{m_1 g - m_2 g}{m_1 + m_2} = \frac{0.100kg \cdot 9.82m/s^2 - 0.015kg \cdot 9.82m/s^2}{0.100kg + 0.015kg} = 7.26m/s^2$$

Percent Error is calculated using the equation

$$(6)\quad \% = \frac{experimental - theoretical}{theoretical} \cdot 100\%.$$

Thus for the first trial the percent was

$$\% = \frac{experimental - theoretical}{theoretical} \cdot 100\% = \frac{7.02 - 7.26}{7.26} \cdot 100\% = -3.3\%.$$

Here the negative simply means the experimental value was smaller then the theoretical a value.

Appendix C: Setup Diagram

Please note the Atwood's machine is connected to a computer with Smart Pulley Software which will automatically measure the acceleration of the two mass system.

Figure 2: Atwood's Machine

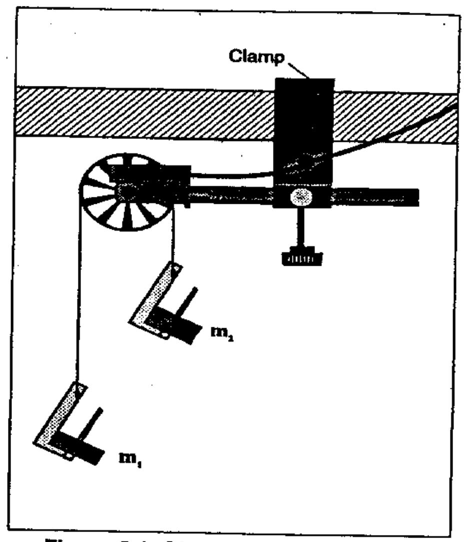

Figure 6.1 Atwood's Machine Setup

Appendix D: Hand Written Data.

Table 2: Handwritten Data

Trial	*m_1*	*m_2*	*Experimental Acceleration*
1	100*	15	7.02
2	90	25	5.65
3	65	50	1.30
4	75	40	2.14
5	60	55	.60

*I could not publish with a handwritten chart, so you will have to use your imagination!